Noise learning in empirical Bayesian source reconstruction algorithms for
electromagnetic brain imaging

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ABSTRACT

Electromagnetic brain imaging is the reconstruction of brain activity from
electroencephalography (EEG) and magnetoencephalography (MEG) data. Robust
estimation of the activity of multiple correlated electromagnetic brain sources
has long been a challenging task, one that is significantly compounded by the
effect of interference from spontaneous brain activity, sensor noise, and other
artifacts. Empirical Bayesian-based algorithms, like Champagne, have been
successful in addressing these issues in a principled fashion. Inherent to the
success of these algorithms is the assumption that an estimate of the noise and
interference can be obtained by a separate "baseline data". However, in many
scenarios, such baseline data is not always available or may be unreliable for
noise estimation. Here, we propose robust methods to estimate heteroscedastic
sensor noise covariance without the need for additional baseline or pre-stimulus
data (Champagne with EM, NL and CB_NL). We demonstrate, both in simulations
and with real data, that noise learning can result in robust reconstruction of
complex brain source activity without the need for baseline data.

MODEL

• Generative data model:
  \[ y(t) = Lx(t) + \epsilon(t) = \sum_{n=1}^{N} L_n x_n (t) + \epsilon(t) \]

  • Where, \( y(t) \in \mathbb{R}^{M \times 1} \) is the output data and \( L = [L_1, \ldots, L_N] \) is the leadfield matrix, \( N \) is the
  number of voxels, \( L_n = [l_{1n}, l_{2n}] \in \mathbb{R}^{M \times 1}, d_n \) is the number of orientations for each voxel.

Champagne algorithm

• The prior distribution for sources:
  \[ p(S) = \prod_{n=1}^{N} \mathcal{N}(s_n(0), \lambda_n^{-1}) = \prod_{n=1}^{N} \mathcal{N}(s_n(0), \lambda_n^{-1} s_n^{T} L_n L_n^{T} s_n) \]

  • where \( a, L_n, \lambda_n \) is the prior precision for the \( n \)-th voxel and \( L_n, \lambda_n \) is a \( d_n \times d_n \) identity
  matrix. Assuming the unknown prior distribution for Gaussian noise \( e : p(e) = \mathcal{N}(0, \Lambda) \),
  therefore the conditional probability \( p(S | Y) = \prod_{n=1}^{N} \mathcal{N}(y(t) | L s_n, \Lambda^{-1}) \)

• The marginal likelihood of the model is convex bounded by the constant function with auxiliary
  variables \( v \in \mathbb{R}^{N} \).\

\[ F(y) = \frac{1}{2} \sum_{n=1}^{N} (y(t) - L s_n)^{T} \Lambda (y(t) - L s_n) + \frac{1}{2} \sum_{n=1}^{N} s_n^{T} \Lambda^{-1} s_n + z^{T} v - z \]

• Update rules:
  \[ s_n(t) \leftarrow v \left( L_n^{T} \sum_{t=0}^{\infty} \Lambda^{T} \Lambda \right)^{-1} y(t) \]
  \[ v_n \leftarrow \left( \sum_{t=0}^{\infty} \Lambda^{T} \Lambda \right)^{-1} \bar{s}_n \]

  • where, \( \bar{s}_n = \Lambda s_n^{T} + \Lambda^{-1} \)

Robust noise learning

• The noise precision \( \Lambda \) can be estimated using EM or convexity-based approaches, assuming
  \( \Lambda^{-1} = diag(\lambda_1^{-1}, \ldots, \lambda_N^{-1}) \), update rules are:

  • Expectation-maximization (EM_NL):
    \[ \hat{\lambda}_n^{t+1} = \frac{1}{L} \sum_{i} (y_i(t) - \hat{\mu}_n(t))^{2} / \hat{\mu}_n \]

  • Convexity-based (CB_NL):
    \[ \hat{\lambda}_n^{t+1} = \frac{1}{L} \sum_{i} (y_i(t) - \hat{\mu}_n(t))^{2} / \hat{\mu}_n \]

RESULTS

Left figure: Aggregate performance in simulations for different noise levels for
Champagne with Noise, Sub and for the noise learning algorithms.
Five randomly point sources are seeded with inter-source
relation coefficient of 0.99. The SNR is set to 3 dB
with Gaussian Noise (blue) and Real
Brain Noise (red). Results are averaged with 50 simulations at
each data point and the error bars show the standard error.

Below figure: Auditory evoked field (AEF) localization results versus number of
tests from one representative subject using
Champagne with Noise Sub (0.01% to 10%), sLORETA, and
Champagne with CB_NL. Champagne with CB_NL is able to
localize the expected bilateral brain activation with focal
reconstructions under even a few trials or even a single trial.
The limited number of trials does not influence the reconstruction results.

CONCLUSIONS

We demonstrate that without any "baseline" data, empirical Bayesian reconstruction algorithms with noise
covariance estimation are robust to the effects of high levels of noise, interference, and highly correlated brain source
activity. We show that noise learning can enable reconstruction of even single trials and demonstrate reconstruction
of resting-state spontaneous brain data. This work is important for demonstrating the reliability and confidence
about high-fidelity reconstructions for MEG/EEG data.